EXECUTING THE FREGEAN PROGRAMME

LECTURE 3

Moreno Mitrović

The HU Lectures on Formal Semantics

- Sets
  - $x \in A$
  - $\varnothing \subseteq A$ (for any A)
  - Set-relations, Venn diagrams
- Functions (blackboard)
- Questions?

FREGE'S CONJECTURE

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RECAP

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∴ How can unsaturated meanings be analysed as sets of ordered pairs? Pairs of what? (Hm.)

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- To understand this, we'll take an excursus.

EXTENSION & FIRST APPLICATION

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(1) a. $x = x$

b. $[\![x]\!]$ = the denotation (meaning) of x

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- We will be interpreting English using **extensional semantics**.

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- (3 min. discussion)

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- Answer: a function from D to $\{0, 1\}$
- Could we write it more formally?

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ii. **Lexicon** (For **terminal nodes only**; we automatically know the denotation type of words.)
iii. **Rules of composition**, a.k.a., rules for 'non-terminal nodes' (We'll need some syntax now.)

Inventory of denotations

- Elements of D, the set of actual individuals.
- Elements of $\{0,1\}$, the set of truth-values.
- Functions from D to $\{0,1\}$.

Lexicon

- Proper names:
  - $[\![$**Ann**$]\!]$ = Ann
  - $[\![$**Bill**$]\!]$ = Bill
- Intransitive verbs:
  - $[\![$**smokes**$]\!] = f : D \mapsto \{0,1\}$
    For all $x \in D$, $f(x) = 1$ iff x smokes
  - $[\![$**works**$]\!] = f : D \mapsto \{0,1\}$
    For all $x \in D$, $f(x) = 1$ iff x works

EXTENSION & FIRST APPLICATION

Some syntax

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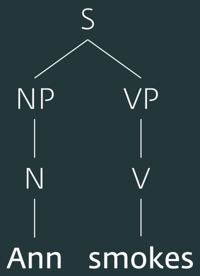
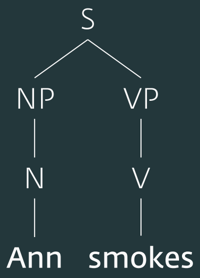


SOME SYNTAX

- A sentence **S** comprises a subject, which is a Noun Phrase **NP**, and a verb, which is actually a Verb Phrase **VP**.
- Every phrase has a head, so **NP** contains a noun head, **N**, and the **VP** contains a verb head, **V**.
- That's the syntax we need for now.

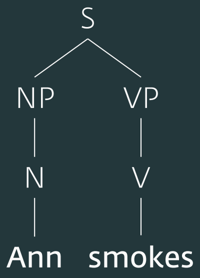


- Let's introduce three simple **structural relations**:

motherhood α is β's **mother** (=mother node) if α immediately dominates β. List some motherhood relations.

daughterhood β is α's **daugher** (=daughter node) if α immediately dominates β. List some motherhood relations.

sisterhood α is β's **sister** (and vice versa) if both α and β are immediately dominated by γ. List the one sisterhood relation.



- And two more pairs of concept:

non/terminal node has (no) daughters.

non-branching node has (no) sisters.

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(4) $[\![\text{Ann smokes}]\!] = \left[\!\!\left[\begin{array}{ccc} & \text{S} & \\ / & & \backslash \\ \text{NP} & & \text{VP} \\ | & & | \\ \text{N} & & \text{V} \\ | & & | \\ \text{Ann} & & \text{smokes} \end{array}\right]\!\!\right]$

- For now, there are only two rules for interpreting trees, depending on whether the sub/tree is non/branching:

Rule #1

In a non-branching node, the denotation of the daughter is inherited by the mother.

$$\begin{array}{c} [\![\alpha]\!] \\ | \\ [\![\beta]\!] \\ | \\ [\![\gamma]\!] \end{array}$$

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$$\begin{tikzcd}[column sep=small]
& \llbracket\alpha\rrbracket \arrow[dl, dash] \arrow[dr, dash] \arrow[dl, bend right=60, dashed] & \\
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$$\begin{tikzcd}[row sep=small]
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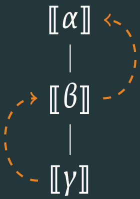

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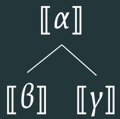

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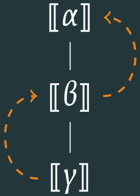

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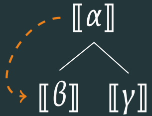

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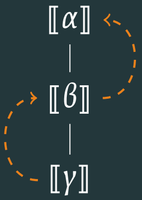

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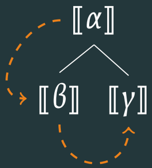

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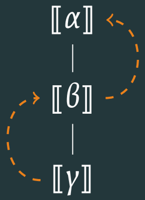

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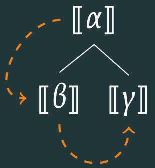

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- **The extension of of an intransitive verb like "smoke", then, should be a function from individuals to truth-values.**


S
NP
VP
N
V
Ann
smokes
=
⟦S⟧
⟦NP⟧
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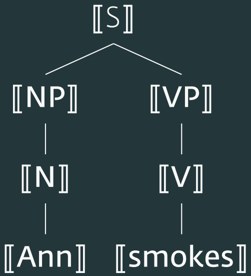
APPLYING THE INTERPRETATION RULES TO TREES

- **Lexicon**: denotation of ⟦Ann⟧



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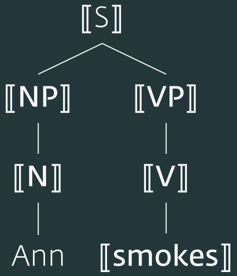

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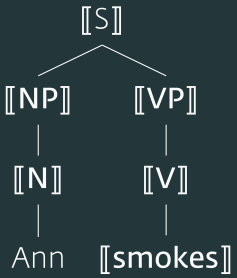

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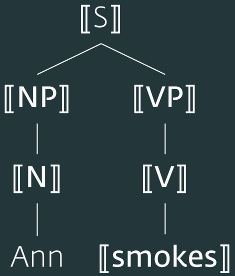

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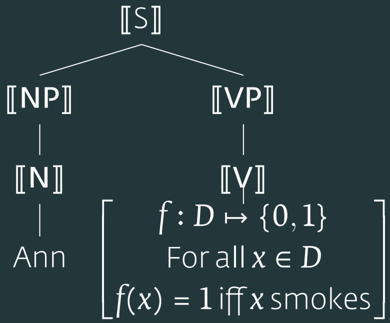

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-

⟦S⟧

Ann

$$\begin{bmatrix} f : D \mapsto \{0,1\} \\ \text{For all } x \in D \\ f(x) = 1 \text{ iff } x \text{ smokes} \end{bmatrix}$$

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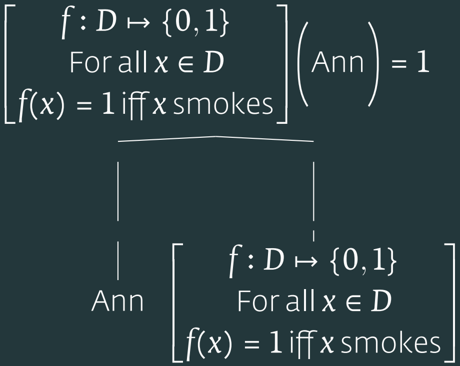
- **Composition rule**: branching nodes as FA at S-level.



EXTENSION & FIRST APPLICATION

BACK TO TRUTH-CONDITIONS

- Suppose Ann, Jan, and Maria are the only individuals in the actual world.
- Ann and Jan are the only smokers.
- The extension of the verb ”smoke” can, in this world, be displayed as follows:

$$[\![\textbf{smokes}]\!] = \begin{bmatrix} \text{Ann} \mapsto 1 \\ \text{Jan} \mapsto 1 \\ \text{Maria} \mapsto 0 \end{bmatrix}$$

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DERIVING TRUTH-CONDITIONS IN AN EXTENSIONAL SEMANTICS

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EXTENSION & FIRST APPLICATION

CHARACTERISTIC FUNCTIONS

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Characteristic function

a. Let A be a set. Then CHAR$_f$, the **characteristic function** of A, is that function f:

$$f(x) = \begin{cases} 1 & \text{for any } x \in A \\ & \end{cases}$$

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b. Let f be a function with range $\{0,1\}$. Then CHAR$_f$, **the set characterised by** f, is $\{x \in D : f(x) = 1\}$

Context

Let our universe contain only three individuals: {Ann, Jan, Maria}. Suppose that Ann and Jan are the only ones who sleep, and Ann is the only one who snores.

Example: set treatment

If intransitive verbs denote sets, then **sleep** and **snore** denote the following:

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(5) a. $[\![\textbf{sleep}]\!] = \{\text{Ann}, \text{Jan}\}$

b. $[\![\textbf{snore}]\!] = \{\text{Ann}\}$

(6) a. $\text{Ann} \in [\![\textbf{sleep}]\!]$

b. $[\![\textbf{snore}]\!] \subseteq [\![\textbf{sleep}]\!]$

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- We will adopt the CHAR$_f$ notation and conception (right column) and drop basic set notation.

	Old system	**New system**
$[\![\mathrm{V_{INTR}} =]\!]$	Set	CHAR$_f$

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⟦V$_{\text{INTR}}$ =⟧	Set	CHAR$_f$
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	Old system	New system
$[\![$V$_{\text{INTR}}$ =$]\!]$	Set	CHAR$_f$
$[\![$**Ann sleeps**$]\!]$ =	Ann $\in$ $[\![$**sleep**$]\!]$	$[\![$**sleep**$]\!]$(Ann) = 1
Set rel.	$[\![$**snore**$]\!]$ $\subseteq$ $[\![$**sleep**$]\!]$	$\{x : [\![\textbf{snore}]\!](x) = 1\} \subseteq \{x : [\![\textbf{sleep}]\!](x) = 1\}$

ADDING TRANSITIVE VERBS

WHAT ABOUT TRANSITIVE VERBS?

(9) Ann smokes.

What about transitive verbs?

(9) Ann smokes.

(10) Ann likes Jan.

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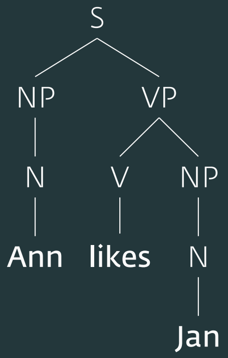
(11)



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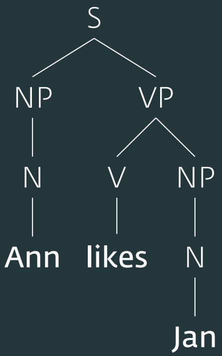

- How do we define the meaning of **likes**, given what we know about the meaning of an intransitive verb like **smokes**?

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- Recall branching-node meaning and the inventory of meanings ...(What's different here?)

Domain of individuals

e is the type of individuals (**e**ntities), where $D_{\mathbf{e}} := D$.

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t is the type of truth values, where $D_{\mathsf{t}} := \{0, 1\}$.

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- **e** and **t** are basic types and correspond to Frege's **saturated meanings**.
- What, then, are **unsaturated meanings**?

- They are of derived types for various functions.

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Domains of derived types

a. $D_{\langle e,t\rangle} := \{f : f \text{ is a function from } D_e \mapsto D_t\}$

b. $D_{\langle e,\langle e,t\rangle\rangle} := \{f : f \text{ is a function from } D_e \mapsto D_{\langle e,t\rangle}\}$

c. ...

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Semantic denotation domains

a. $D_{\mathsf{e}} := D$ (**the set of** individuals)

b. $D_{\mathsf{t}} := \{0, 1\}$ (**the set of** truth-values)

c. For any semantic types σ and τ, $D_{\langle \sigma, \tau \rangle}$ is **the set of** all functions from D_σ to D_τ.

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  - type $\langle \mathbf{e}, \langle \mathbf{e}, \mathbf{t} \rangle \rangle$ (example: **transitive Vs**)
  - type $\mathbf{t}$ (example: **sentences**)

- We've covered a conceptually vast, yet relatively simple, metalinguistic system with(in) which we can analyse meanings.
- We now have **two more technical matters** to address:
    - One will decompose 2-place functions (=transitive Vs) and make sense of them in terms of the system we've been developing.
    - Another will simplify the technical issues with the way we've been writing down functions. It will make life easier. And it makes much sense.

SCHÖNFINKELISATION

- We need a bit more maths to synthesise the last portion of slides and understand trans-Vs as 2-place functions.
- Recall our three general assumptions:

Binary branching In the syntax, trans-Vs combine with the direct object to form a VP, and VPs combine with the subject to form a sentence.

Locality Semantic interpretation rules are local: the denotation of any non-terminal node is computed from the denotation of its daughter nodes.

Frege's conjecture Semantic composition is functional application.

Example

Let our domain D contain just the three goats Sebastian, Dimitri, and Leopold. Sebastian is the biggest and Leopold the smallest. The relation "is-bigger-than" is then the following set of ordered pairs:

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(16) $$R_{\text{BIGGER}} = \left\{ \begin{array}{c} \langle \text{Sebastian, Dimitri} \rangle, \\ \langle \text{Sebastian, Leopold} \rangle, \\ \langle \text{Dimitri, Leopold} \rangle \end{array} \right\}$$

- There is a correspondence between sets and their characteristic functions.
- What is the *functional version* of R_{BIGGER}? (3mins)

- The resulting function f_{BIGGER} is a **2-place function**.
- Moses Schönfinkel, a logician, showed that n-place functions are **reducible to 1-place function**.
- This reduction is *Schönfinkelisation*.

$$f_{\text{BIGGER}} = \begin{bmatrix} \langle \text{L}, \text{S} \rangle \mapsto 0 \\ \langle \text{L}, \text{D} \rangle \mapsto 0 \\ \langle \text{L}, \text{L} \rangle \mapsto 0 \\ \langle \text{S}, \text{L} \rangle \mapsto 1 \\ \langle \text{S}, \text{D} \rangle \mapsto 1 \\ \langle \text{S}, \text{S} \rangle \mapsto 0 \\ \langle \text{D}, \text{L} \rangle \mapsto 1 \\ \langle \text{D}, \text{S} \rangle \mapsto 0 \\ \langle \text{D}, \text{D} \rangle \mapsto 0 \end{bmatrix}$$

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- f'_{BIGGER} is a function that applies to the first arg. and yields a function that applies to the second arg.
- When applied to Leopold, it yields a function that maps any goat to 1 **if it is smaller than Leopold**.

- We could also do it the other way round: have the function apply to the second argument and yield a function that applies to the first argument.
- Think of our syntactic tree.
- When applied to Leopold, let f'' yield a function that maps any goat to 1 **if it is bigger than Leopold**.

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- We add some very special operators, **lambdas** (λ), to our system in order to simplify it.
- **The λ operator applies to a function in order to describe it.**

- Before we move onto this, let's recall our $\textsc{char}_f$-notation for intransitive verbs.

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- What about transitive verbs?

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(18) $[\![\textbf{Ann loves Jan}]\!] =$ (two notations)

a. $[\![\textbf{loves}]\!](\text{Ann})(\text{Jan}) = 1$ (iff Ann actually loves Jan)

b. $[\![\textbf{loves}]\!](\text{Ann}, \text{Jan}) = 1$ (iff Ann actually loves Jan)

- Imagine we were interpreting an expression containing just the two words: noun **Maggie** and verb **love(s)**
  - We first need to construct a tree. In our case, there are two possible trees since something is missing.

(19)

(20)

$\boxed{\lambda x}.\mathrm{LOVES}(\mathrm{Mary}, \boxed{x})$

denotes the characteristic function of the set of individuals that **Maggie loves**.

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(22) $\lambda x.[\![\text{smokes}]\!](x) = \ldots$

- The last notation can be read 'if there was an x, $[\![\text{smoke}]\!]$ could be true.'

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For example

Expression $\textsc{bored}(x)$ **denotes the result of applying** the function denoted by **bored** to the value of x.

Another example

(23) $\left[\lambda x.\textsc{loves}(\text{Maggie})(x)\right]$

Another example

(23) $\left[\lambda x.\textsc{loves}(\text{Maggie})(x)\right](\text{Bill})$

- 23 denotes the result of applying the function **is loved by Maggie** to **Bill**.
- This is then equivalent to (24)

(24) $\textsc{loves}(\text{Maggie})(\text{Bill})$

- where **Bill** replaced the placeholder x.

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- where **Bill** replaced the placeholder x.
- This 'conversion' process is known as **β-conversion** or **β-reduction**.

- We all remember formulae like (25) from high school.

(25) $f(x) = x + 7$

a. Now let $x = 5$.

b. Then we have:
$f(x) = x + 7 \rightsquigarrow f(5) = 5 + 7$

- (25) is the same as (27)

(26) a. $f(x) = x + 7 \rightsquigarrow \lambda x.x + 7$

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λ-abstraction with numbers: a sketch

(27) a. $f(x) = x + 7 \rightsquigarrow \lambda x.x + 7$

b. $[\lambda x.x + 7]$

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- That's all λ-abstraction is:
  - abstraction with a λ-clause specifies the conditions under which the **value description** (27a)
  - β-reduction (β-conversion), **reduces** or **converts** the variable x into whatever value we feed it – in our case, number 5.

QUESTIONS?

EXERCISES

(28) $29 \in \{x \in \mathbb{N} : x \neq 0\}$ iff $29 \neq 0$

(29) Massachusetts $\in \{x \in D :$ California is a western state$\} = D$ iff California is a Western state.

(30) $\{x \in D :$ California is a western state$\} = D$ if California is a western state.

(31) $\{x \in D :$ California is a western state$\} = \varnothing$ if California is not a western state.

(32) $\{x \in \mathbb{N} : x \neq 0\} = \{y \in \mathbb{N} : y \neq 0\}$

(33) $[\lambda x \in D[\lambda y \in D[\lambda z \in D.z \text{ introduced } x \text{ to } y]]](\text{Ann})(\text{Sue})$

(34) $[\lambda x \in \mathbb{N}[\lambda y \in \mathbb{N}.y > 3 \text{ and } y < 7](x)]]]$