INTRODUCTION & CORE CONCEPTS

ENGLISH SEMANTICS • LECTURE 1

Moreno Mitrović

October 4, 2014 ($\frac{1}{2}$)

The Saarland Lectures on Formal Semantics

THE COURSE MANUAL

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LOGISTICS

COURSE LOGISTICS

- This is a **bootcamp course**.
- You will receive 14 lectures in a space of 7 days.
  - Every day @ $\begin{cases} 12{:}00\text{--}13.30 \\ 14{:}00\text{--}15{:}30 \end{cases}$ (C5-3 / U13)
- Course material, including slides and problem sets, will be deposited on my website: `http://mitrovic.co/teaching`
- **Attendance is extremely important**: we will build our analysis incrementally (missing one lecture will set you back).

THE COURSE MANUAL

MATERIAL

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- We will use Heim & Kratzer [HK], the key semantics textbook out there:

  Heim, I. & A. Kratzer. 1998. *Semantics in Generative Grammar.* Oxford: Blackwell.
- This textbook is generally used in post-graduate courses, so the standard is high but so is the pay-off.
- You are also advised to resort to another great textbook, which may be a stepping stone to HK:

  Coppock, E. 2016. *Semantics Boot Camp.* Ms.
  - Accessible here:

    `http://eecoppock.info/semantics-boot-camp.pdf`
- We will also use a piece of software to help us practice semantic analysis:

  Champollion, L., *et al.* 2016. *The Lambda Calculator.*
  - Accessible here: `http://lambdacalculator.com`

THE COURSE MANUAL

ASSESSMENT PROTOCOL

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- The point of this course is to teach you an analytical skill (there will be **no memorising – just understanding**).
- Conversely, the examination protocol is designed to assess just that using
  - **two closed problem sets** (short assignments you solve under examination conditions after lecture)
  - **one portfolio of open problem sets** (a set of assignments you solve at home)

TYPE	DEADLINE/TIME	WEIGHT
interim closed problem set	Oct 7	30%
final closed problem set	Oct 12	30%
portfolio of open problem sets	Oct 12 [TBC]	40%

QUESTIONS BEFORE WE MOVE ONTO SEMANTICS?

TRUTH-CONDITIONAL SEMANTICS & THE FREGEAN PROGRAMME

LINGUISTICS & SEMANTICS: WHY DO IT? (HEIM'S TALK)

- Language is probably the most **striking manifestation of human intelligence**.
- Not only a **tool for sharing thought** (communication), but also a powerful **tool for thought itself**.
- Linguists, and other cognitive scientists, aim to **discover the mental data structures and algorithms** that are involved in thinking, reasoning, and understanding/using language.
    - How can a sequence of sounds give rise to a sequence of thought? How can it trigger a chain of deduction?
    - There have to exist symbolic structures.

SEMANTICS: THE STUDY MEANING

- Semantics is the study of meaning
    - Philosophical approaches
    - Linguistic approaches
- If **meaning** is the **object of our study**, we need to know **what it is**, or **what it means**
    - (The fact that it sounds funny is curious in itself, and the idea of *the meaning of meaning* alone shows a philosophical problem).

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TRUTH-CONDITIONAL SEMANTICS & THE FREGEAN PROGRAMME

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- The magical mystery: we can understand sentences without knowing whether they're true! (Wittgenstein)
- The second property: meaning is compositional.

Principle of Compositionality

The meaning of a sentence is computed from the meaning of its parts.

Our goal

to break down sentences into their parts and consider the contribution of each part to the truth-conditions of the whole.

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THE FREGEAN PROGRAMME

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- This insight and principle of compositionality is due to Gottlob Frege:

  It is astonishing what language accomplishes. [Communication] would not be possible if we could not distinguish parts in the thought that correspond to parts of the sentence, so that the construction of the sentence can be taken to mirror the construction of through …The question now arises how the construction of the thought proceeds, and by what means the parts are put together so that the whole is something more than the isolated parts.

COMPOSITIONALITY

In my essay "Negation," I considered the case of a thought that appears to be composed of one part which is in need of completion or, as one might say, unsaturated, and whose linguistic correlate is the negative particle, and another part which is a thought. We cannot negate without negating something, and this something is a thought. Because this thought saturates the unsaturated part or, as one might say, completes what is in need of completion, the whole hangs together. And it is a natural conjecture that logical combinations of parts into a whole is always a matter of saturating something unsaturated. ☆

☆ Frege, G. 1923--6. Logische Untersuchungen. Dritter Teil: Gedankengefüge. *BzPdDI*. 3:36--51.

MEANING = TRUTH-CONDITIONS + COMPOSITIONALITY

- So negation is unsaturated, i.e., it is in need of completion.
- A proposition without negation is saturated, it is a complete semantic unit with its own truth-conditions.

(2) a. Trump is an idiot.
    b. Trump is not an idiot.

- A sketch of meanings for (2):

(3) a. "(2a)" is true iff (2a).
    b. i. "(2b)" is true iff (2b).

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       ii. "(2b)" is true iff (2a) is false.

MEANING = TRUTH-CONDITIONS + COMPOSITIONALITY

- Meaning: saturated or unsaturated?

	SAUTRATED	UNSAUTRATED
'not'		
'Trump'		
'every'		
'fall'		

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- (How else could we understand saturation/completeness of meaning?)

MORE ON MEANING SATURATION

Statements in general, just like equations ..., can be imagined to be split up into two parts; one complete in itself, and the other in need of supplementation, or "unsaturated." Thus, e.g., we split up the sentence

"Caesar conquered Gaul"

into "Caesar" and "conquered Gaul." The second part is "unsaturated" -- it contains an empty place; only when this place is filled up with a proper name, or with an expression that replaces a proper name, does a complete sense appear.

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ZOOMING OUT: TOWARDS A METALANGUAGE

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    - Set theory and Functions (today)

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    - Set theory and Functions (today)
    - Type theory (maybe tomorrow?)

METALANGUAGE

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SET THEORY

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- Set theory is a **theory of sets**. Go figure.

Set

A set is a **collection** of objects which are called the **members** or **elements** of that set.

(We will use upper-case letters (A) or curly brackets ($\{\ \}$), to refer to sets, and lower-case letters (x) to refer to elements.)

SOME SET RELATIONS & SET-THEORETIC CONCEPTS

MEMBERSHIP if object x belongs to set A, we will write $x \in A$, which reads "x is an elements of A". (Conversely, if x is not in A, then $x \notin A$)

EMPTINESS If a set has no elements, it is an empty set. We will refer to this set using the symbol $\varnothing$.

EQUALITY If two sets comprise of the same elements, e.g., $A = \{x, y, z\}$, $B = \{x, y, z\}$, then $A = B$

SUBSETHOOD If all the members of one set (A) are also members of another (B), then the former is a subset of the latter, i.e. $A \subseteq B$ ("A is a subset of B"). (Conversely, $B \supseteq B$, "B is a superset of A".)

SOME SET RELATIONS & SET-THEORETIC CONCEPTS (CONT.)

INTERSECTION An intersection of two sets, $A \cap B$, will only contain elements who are members or both A and B.

UNION A union of two sets, $A \cup B$, will contain elements who are members of A or B or $A \cup B$.

DIFFERENCE A difference of two sets, $A - B$ or $A \setminus B$ ("A without B"), contains members in A and no members in B (subtraction-like).

COMPLEMENTATION A complement of a single set, A^C (or, A^-, or $\bar{A}$), contains everything not in A.

UNIVERSE Universe, U, is a superset of all sets, i.e., a collection of everything. (E.g., the complement of set A ($\bar{A}$) is $U \setminus A$.)

SPECIFYING SETS: DEFINITION BY LISTING

- How do we specify what elements belong to a set?
- One way is to list those elements.

(4)
a. $A = \{a, b, c\}$
b. $B = \{1, 2, 3, \ldots\}$
c. $C = \{\bigstar, \blacklozenge, \bullet\}$
d. ...

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- Not terribly useful, at least for our purposes.

SPECIFYING SETS: DEFINITION BY ABSTRACTION

- Another way is to specify a condition which is to be satisfied by all and only the elements of the set we are defining.

(5) a. Let A be set of all cats.

b. Let A be that set which contains exactly those x such that x is a cat.

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A "the set of all x such that x is a cat"

SPECIFYING SETS: DEFINITION BY ABSTRACTION

- The element x in (5c), repeated below as (6), does not stand for a particular object (=CONSTANT), rather it is a VARIABLE (=a 'place-holder').
- To determine the membership of the set A, one has to plug in the the names of actual objects ('cats') for the "x" in the condition "x is a CAT"
- If we want to know whether Fido $\in A$, we must consider the statement 'Fido is a CAT'
  - If this statement is true, then Fido $\in A$.
  - If this statement is false, then Fido $\notin A$

(6) $A = \{x : x \text{ is a CAT}\}$

a. $A = \{x : x \text{ is a CAT}\}$ x is a **variable**

b. $A \neq \{x\}$ x is a **constant**

METALANGUAGE

FUNCTIONS

FUNCTIONS

- Recall that Frege took **unsaturated meanings** to be **functions**.
- We now know about sets. Well, functions are sets, too. (Particular kinds of sets.)
- Functions are like factory machines: they takes material (input) and deliver a product (output).
- If we have two objects, x and y, we can construct from them an **ordered pair** which we notate as $\langle x, y \rangle$.

(7) **However!**

a. $\{x, y\} = \{y, x\}$ order **doesn't matter**

b. $\langle x, y \rangle \neq \langle y, x \rangle$ order **matters**!

FUNCTIONS AS RELATIONS

- A relation is a set of ordered pairs.
- Functions are a special kind of relation: in a function, the second member of each pair, e.g., $\langle x, \boxed{y} \rangle$, is UNIQUELY DETERMINED by the first, i.e., $\langle \boxed{x}, y \rangle$

Relation

A relation f is a function iff it satisfies the following condition:

For any x : if there are y and z such that $\langle x, y \rangle \in f$ and $\langle x, z \rangle \in f$, then $y = z$.

FUNCTIONS: THEIR DOMAIN AND RANGE

- Every function has a DOMAIN and a RANGE (both of which are sets).

Domain and Range

Let f be a function.
Then the **domain** of f is $\{x :$ there is a y such that $\langle x, y \rangle \in f\}$,
and the **range** of f is $\{x :$ there is a y such that $\langle y, x \rangle \in f\}$

- Another way to notate a function as a mapping from its domain (A) to its range (B):
  - $f : A \mapsto B$
- There are also other notations:
  - $f(x)$ ("f **applied to** x", or "f of x")
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SPECIFYING FUNCTIONS: DEFINITION BY LISTING

- Just like we did with sets, we can define functions in many ways.
- One way is to list the f's elements (=ordered pairs).
- Here are two equivalent list notations:

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- Here are two equivalent list notations:

(8) a. $f := \{\langle a, b\rangle, \langle c, b\rangle, \langle d, e\rangle\}$

b. $$f := \begin{bmatrix} a \mapsto b \\ c \mapsto b \\ d \mapsto e \end{bmatrix}$$

c. Let f be a function with domain $\{a, c, d\}$, such that

  i. $f(a) = b = f(c)$

  ii. $f(c) = b = f(a)$

  iii. $f(d) = e$

SPECIFYING FUNCTIONS: DEFINITION BY ABSTRACTION

- Functions with large or infinite domains are better defined in abstract form, by specifying the condition that is to be met by each ⟨argument-value⟩ pair.
- You're probably more familiar with this notation from your high-school math classes.

(9) $f : \mathbb{N} \mapsto \mathbb{N}$, and for every $x \in \mathbb{N}$, $f(x) = x + 1$

(where $\mathbb{N}$ is the set of all natural numbers)

- Once we equip ourselves with some more technical tools, we will soon introduce an even more concise notation for such functions.

HOMEWORK & FIRST PROBLEM SET

- It is very important we all understand these formal preliminaries.
- To this end, a problem set on set theory and functions is due TOMORROW.
- Downloadable from `mitrovic.co/teaching` → filed under 'Saarland' (second tab)

QUESTIONS?